4.1) 1d $f(x)=x^4$; $g(x)=\sqrt[4]{x}$

If f and g are inverses [(x,y) in "f" then (y,x) in "g"]

domain of f(x): $(-\infty,\infty)$ $f(-3)=81$ $(-3,+81)$

domain of g(x): $[0,\infty)$ $(81,-3)$

range of f(x): $[0,\infty)$

range of g(x): $[0,\infty)$

2 $f(x)=x^4$ $g(x)=\sqrt[4]{x}$

$\Rightarrow f(g(x))=(\sqrt[4]{x})^4=x$, for every x in domain g

$g(f(x))=\sqrt[4]{(x^4)}=|x|$

Not $=x$ for every x in domain of f

remember how to "solve" for inverses

$f(x)=x^4$

replace function name with "y"

$y=x^4$

switch x's and y's

$x=y^4$

solve for y

$f(x) = \{(0,2),(1,4),(3,7)\}$

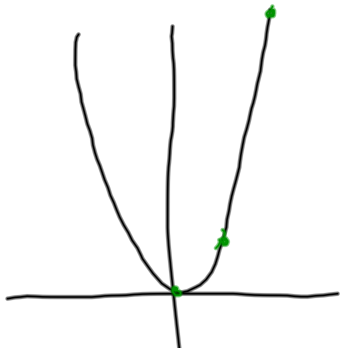
domain: $\{0,1,3\}$

range: $\{2,4,7\}$

$f^{-1}(x) = \{(2,0),(4,1),(7,3)\}$

domain: $\{2,4,7\}$

range: $\{0,1,3\}$



$\{(0,0),(1,1),(2,16),(-1,1),(-2,16),(\frac{1}{2},\frac{1}{16}),(\frac{1}{3},\frac{1}{81}),\ldots\}$

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4.1.1
pg 242
$2 \cdot 11^2$

* $f(x)$, $g(x)$ are functions.
$f(x)$ and $g(x)$ are inverse functions
and we write $g(x) = f^{-1}(x)$
[[and, obviously, $f(x) = g^{-1}(x)$]]
if (& the other way)
$f(g(x)) = x$ for every x in the domain of $g(x)$
$g(f(x)) = x$ for every x in the domain of $f(x)$

Remember that composition can't make domains LARGER!

2

CAN COMPOSITION MAKE DOMAINS SMALLER?

$f(x) = \frac{1}{x^2}$ $\cancel{0}$ $g(x) = \sqrt[4]{x}$ 0

domain: $x \neq 0$
range: $(0, \infty)$

domain: $[0, \infty)$
range: $[0, \infty)$

$f(g(x))$

domain
f $(-\infty, 0) \cup (0, \infty)$
g

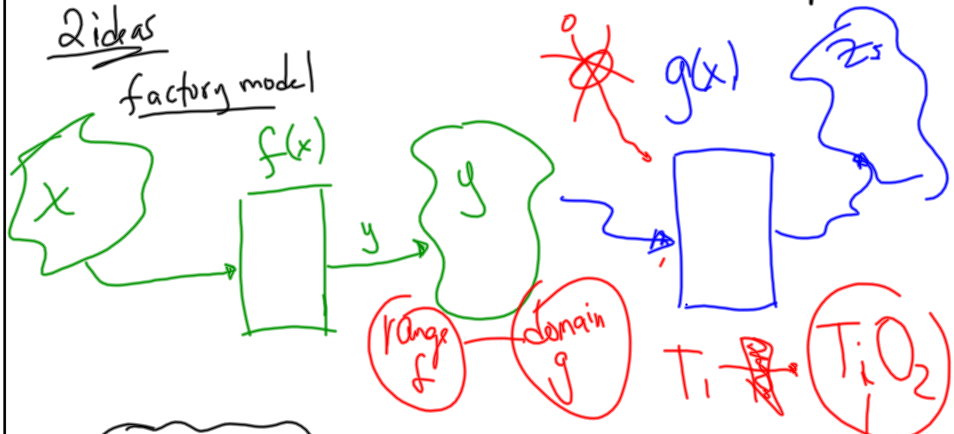
range: $[0, \infty)$

inverse trig

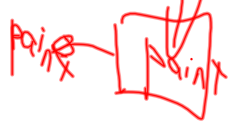
What is this composition thing?

2 ideas

factory model



Substitution model



$f(x) = \frac{1}{x^2}$; $g(x) = \sqrt[4]{x}$

$f\circ g(x) = f(g(x))$ = the result when $g(x)$ [[the function]] is substituted in for x in the rule of $f(x)$.

$$f(g(x)) = \frac{1}{(g(x))^2} = \frac{1}{(\sqrt[4]{x})^2} = \frac{1}{\sqrt{x}}$$

you can talk about $f\circ g(x) = f(g(x))$

AND $g\circ f(x) = g(f(x))$

In general [[i.e. usually]] these are different.

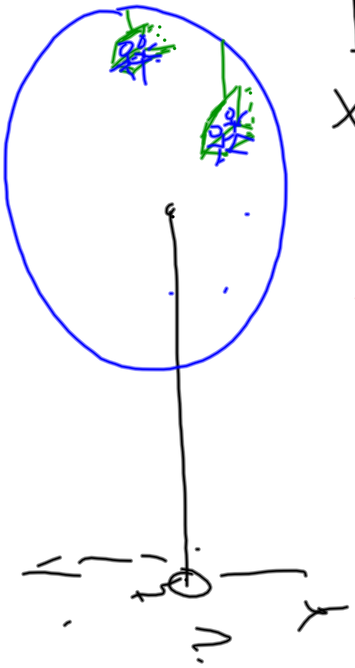
but if $f = g^{-1}$ then

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

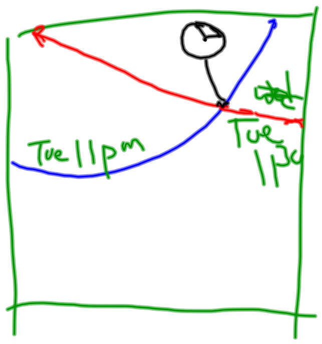
Parametric Equations

You are still 'graphing' (x, y) pairs,

but
x and y are given by
functions of t (usually time).

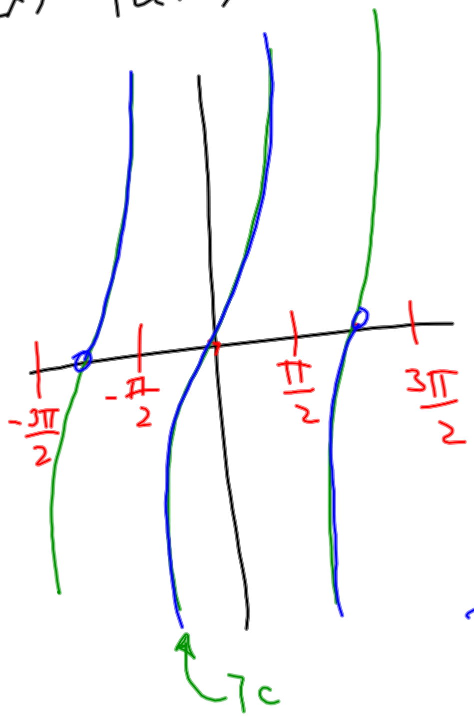


$x = A\cos(t) + h$
$y = A\sin(t) + h$



7) a) $f(x) = \tan x$

a) $\tan x = \frac{\sin x}{\cos x}$



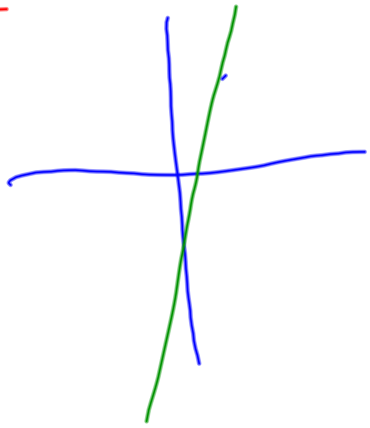
7b) $\tan x$,

$-\pi < x < \pi$

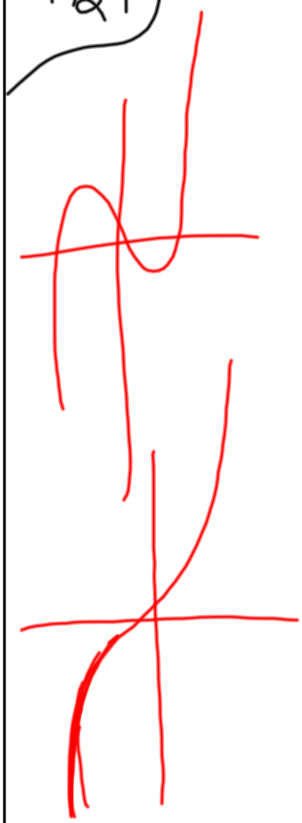
$x \neq \frac{\pi}{2}, -\frac{\pi}{2}$

7c) $\tan x$,

$-\frac{\pi}{2} < x < \frac{\pi}{2}$



7b4)



$p(x) = x^3 - 3x^2 + 3x - 1 = (x-1)^3$

$p^{-1}(x)$:

$y = (x-1)^3$

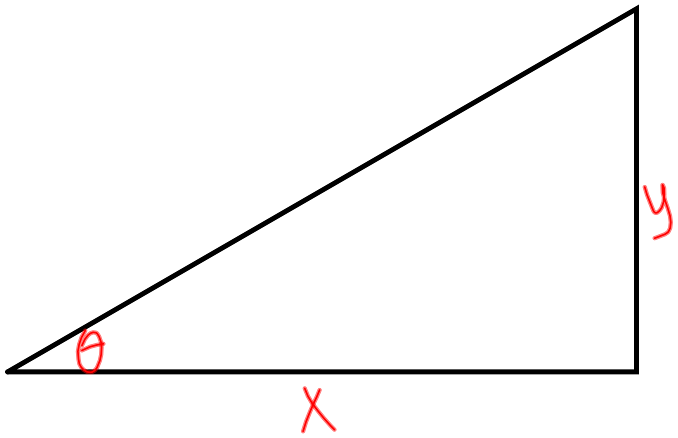
$x = (y-1)^3$

$\sqrt[3]{x} = y - 1$

$\boxed{\sqrt[3]{x} + 1 = y}$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$



$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \arctan\left(\frac{y}{x}\right)$ * careful domain considerations

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